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Fund of Hedge Funds Portfolio Selection: A Multiple Objective Approach

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Abstract

This paper incorporates investor preferences into a Polynomial Goal Programming (PGP) optimization function. This approach allows us to solve for multiple competing (and often conflicting) hedge fund allocation objectives within the mean-variance-skewness-kurtosis framework. Our empirical analysis shows that equity market neutral funds and global macro funds have predominant roles in optimal fund of hedge funds portfolios. Specifically, equity market neutral funds are risk and kurtosis reducers while global macro funds are skewness enhancers.

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1 Introduction

Because hedge fund returns exhibit significant skew and kurtosis, standard mean-variance portfolio analysis is inadequate for analyzing this rapidly growing industry. In this paper, we use Polynomial Goal Programming (PGP) to incorporate investor preferences for higher moments into fund of hedge funds portfolio optimization decisions. PGP enables us to solve for multiple, often conflicting, optimization objectives and to show how changing investor preferences can lead to dramatically different asset allocation decisions across hedge fund strategies.

Jean (1971) and Scott and Horvath (1980) show that if investors' utility functions are assumed to be of higher order than quadratic, portfolio analysis and risk management need to be extended to a higher-moment framework. Research to-date, including Amenc and Martellini (2002), Barès et al. (2002), Cvitanić et al. (2003), Schneeweis and Spurgin (1999), Terhaar et al. (2003), and Morton et al. (2003), however, largely concentrates on hedge fund allocation within mean-variance opportunity sets. Applications of higher moment analysis on hedge fund portfolios are emerging, but mainly aim to identify or confirm the benefit of including hedge funds into stock/bond portfolios (see, for example, Hagelin and Pramborg (2003) and Popova et al. (2003)).

A traditional approach to portfolio optimization is to apply a Taylor series expansion to investor utility. Tsiang (1972) shows that the set of utility functions displaying the desired attributes are the negative exponential function $U(w) = B\,(1 - \exp(-\alpha w))$, the family of constant elasticity power utility functions $U(w) = \frac{1}{1-a} w^{1-a},\ a > 0$, and the logarithmic utility function $U(w) = \log(w)$. Variants of the Taylor series expansion approach are used by Amenc and Martellini (2003), Barès et al. (2002) and Popova et al. (2003) to examine hedge fund portfolio selection. This approach, however, has severe limitations. Negative exponential utility functions, for example, display constant absolute risk aversion, which is not practical in the real world. Power utility functions exhibit decreasing absolute risk aversion which implies that in some range a risk-asset portfolio is an inferior good (see Lai (1991) and Kraus and Litzenberger (1976)). Furthermore, hedge fund investors are mainly accredited wealthy investors and institutional investors, whose heterogeneous preference profiles are difficult to be encompassed by a single utility function. This situation differs from that faced by the typical mutual fund or pension fund which has a relatively homogeneous group of retail investors as their main investor base.

This is where our research fits in. Within the mean-variance-skewness-kurtosis framework, we solve the task of multiple conflicting and competing hedge fund allocation objectives such as maximizing expected return and skewness and minimizing risk and kurtosis simultaneously, by construction of a PGP problem into which the specific investor's personal objectives are incorporated. Our contribution not only lies in pioneering optimal hedge fund portfolio selection into higher moment framework and offering explicit solutions to optimal hedge fund portfolio design, in the context of funds of hedge funds. It also lies in augmenting the dimensionality of PGP portfolio selection—from mean-variance-skewness to mean-variance-skewness-kurtosis—to incorporate more complete information for portfolios with non-normal returns. PGP helps us provide guidance on optimal fund of hedge funds allocation decisions such as: (i) which fund strategies should be included; and (ii) how much capital should be allocated to each strategy.

The remainder of the paper is organized as follows. The next section formulates an ideal hedge fund portfolio selection within mean, variance, skewness and kurtosis as a solution for a competing and conflicting multiple objective problem. Section 3 discusses hedge fund classification and the data we use. Section 4 provides empirical results. Section 5 concludes. The procedure for unsmoothing the data is outlined in the appendix.

2 PGP for portfolio selection within four-moment framework

PGP was first introduced by Tayi and Leonard (1988) to facilitate bank balance sheet management with competing and conflicting objectives. It has subsequently been used by Lai (1991), Chunhachinda, et al. (1997), Sun and Yan (2003), Prakash et al. (2003), and others, to solve portfolio selection with skewness. It can incorporate investors' preference between conflicting goals, from which portfolio selection with skewness is determined. Our contribution is to augment the dimension of portfolio selection in PGP from mean-variance-skewness to mean-variance-skewness-kurtosis, thereby incorporating more complete information for non-normal returns.

The portfolio selection problem may be formulated as a multiple objective programming problem:

$$\begin{aligned}
\text{Maximize} \quad & Z_1 = X^\top \tilde{R}, && (1)\\
\text{maximize} \quad & Z_3 = E[X^\top(\tilde{R} - \hat{R})/(X^\top V X)^{1/2}]^3, && (2)\\
\text{minimize} \quad & Z_4 = E[X^\top(\tilde{R} - \hat{R})/(X^\top V X)^{1/2}]^4 - \frac{3(T-1)^2}{(T-2)(T-3)}, && (3)\\
\text{subject to} \quad & X^\top V X = 1. && (4)
\end{aligned}$$

where, $X^\top = (x_1, x_2, \cdots, x_n)$ and x_i is the percentage of wealth invested in the ith risky asset (in our case, hedge fund strategy); T is the number of observations in one time series; portfolio mean return is Z_1, variance is $X^\top V X$, skewness is Z_3, and excess kurtosis (referred to henceforth as simply kurtosis) is Z_4.

Given an investor's preferences among objectives, a PGP can be expressed instead as:

$$\begin{aligned}
\text{Minimize} \quad & Z = (1 + d_1)^\alpha + (1 + d_3)^\beta + (1 - d_4)^\gamma && (5)\\
\text{subject to} \quad & X^\top \tilde{R} + d_1 = Z_1^* && (6)\\
& E[X^\top(\tilde{R} - \hat{R})/(X^\top V X)^{1/2}]^3 + d_3 = Z_3^* && (7)\\
& E[X^\top(\tilde{R} - \hat{R})/(X^\top V X)^{1/2}]^4 - \frac{3(T-1)^2}{(T-2)(T-3)} + d_4 = Z_4^* && (8)\\
& d_1,\ d_3 \geq 0 && (9)\\
& d_4 \leq 0 && (10)\\
& X^\top V X = 1 && (11)
\end{aligned}$$

where α, β and γ are the nonnegative investor-specific parameters representing the investor's subjective degree of preferences on the mean, skewness and kurtosis of the portfolio return; Z_1^* is the mean return for the optimal mean-variance portfolio with unit variance; Z_3^* is the skewness value of the optimal skewness-variance portfolio with unit variance; Z_4^* is the kurtosis value of the optimal kurtosis-variance portfolio with unit variance. Since mean return (skewness) for an optimal mean-variance-skewness-kurtosis should be less than the expected return (skewness) for an optimal mean-variance portfolio (skewness-variance portfolio), d_1 and d_3 represent positive deviations from Z_1^* and Z_3^*. Similarly, since kurtosis for an optimal mean-variance-skewness-kurtosis portfolio should be greater than the kurtosis value for an optimal kurtosis-variance portfolio, d_4 represents the negative deviation from Z_4^*. The specification of our objective function in (5) ensures that it is monotonically increasing in d_1 and d_3 and monotonically decreasing in d_4, even for deviation values where $d_1, d_3 \in \{0, 1\}$ and $d_4 \in \{-1, 0\}$.

As explained in Lai (1991), solving the multiple objective PGP problem involves a two-step procedure. First, the optimal values of Z_1^*, Z_3^*, and Z_4^*, for the expected return, skewness and kurtosis, respectively, are obtained under the unit variance restriction. Then, these optimal values are substituted into the conditions (6), (7), and (8), and the minimum value of (5) is found for a given set of investor preferences $\{\alpha, \beta, \gamma\}$.

3 Strategy classification and data

Hedge fund investment strategies tend to be quite different from the strategies followed by traditional money managers. In principle every fund follows its own proprietary strategy, which means that hedge funds are a very heterogeneous group. It is, however, customary to ask hedge fund managers to classify themselves into one of a number of different strategy groups depending on the main type of strategy followed. We concentrate on seven main classes of funds. The numbers between brackets indicate the estimated market share of each strategy group in terms of assets under management based on the June 2002 TASS asset flows report:

Long/Short Equity (43%): Funds that invest on both the long and the short side of the equity market. Unlike equity market neutral funds (see below), the portfolio may not always have zero market risk. Most funds have a long bias.

Equity Market Neutral (7%): Funds that simultaneously take long and short positions of the same size within the same market, i.e. portfolios are designed to have zero market risk. Leverage is often applied to enhance returns.

Convertible Arbitrage (9%): Funds that buy undervalued convertible securities, while hedging (most of) the intrinsic risks.

Distressed Securities (11%): Funds that trade the securities of companies in reorganization and/or bankruptcy, ranging from senior secured debt to common stock.

Merger Arbitrage (8%): Funds that trade the stocks of companies involved in a merger or acquisition, buying the stocks of the company being acquired while shorting the stocks of its acquirer.

Global Macro (9%): Funds that aim to profit from major economic trends and events in the global economy, typically large currency and interest rate shifts. These funds make extensive use of leverage and derivatives. These are the funds that are responsible for most media attention.

Emerging Markets (3%): Funds that focus on emerging and less mature markets. These funds tend to be long only because in many emerging markets short selling is not permitted and futures and options are not available.

A separate class of funds is formed by so-called funds of funds. These are funds that solely invest in other hedge funds. Some limit themselves to one specific type of hedge fund strategy but most invest across the board. The idea behind funds of funds is to offer investors a hassle-free alternative to constructing a basket of hedge funds themselves. In addition, many claim to add value by employing experienced managers to select funds, to carry out due diligence and to monitor continuously the portfolio.

The database used in this study covers the period June 1994–May 2001 and was obtained from Tremont TASS, which is one of the best known and largest hedge fund databases currently available. Our database includes the Asian, Russian and LTCM crises as well as the end of the IT bubble and part of the bear market that followed. As of May 2001, the database contains monthly net of fee returns on a total of 2183 hedge funds and funds of funds. Reflecting the tremendous growth of the industry as well as a notoriously high attrition rate, only 264 of these funds had seven or more years of data available.

As shown in Amin and Kat (2003), concentrating on surviving funds only will overestimate the mean return on individual hedge funds by around 2% as well as introduce significant biases in estimates of the standard deviation, skewness and kurtosis. To avoid this problem we decide not to work with the raw return series of the 264 survivor funds but instead to create 358 seven-year monthly return series by, starting off with the 358 funds that were alive in June 1994, replacing every fund that closed down during the sample period by a fund randomly selected from the set of funds alive at the time of closure following the same type of strategy and of similar size and age.

This replacement procedure implicitly assumes that in case of fund closure investors are able to roll from one fund into the other at the reported end-of-month net asset values and at zero additional costs. This underestimates the true costs of fund closure to the investor. First, when a fund closes shop its investors have to look for a replacement investment. This search takes time and is not without costs. Second, investors may get out of the old and into the new fund at values that are less favorable than the end-of-month net asset values contained in the database.

As hedge funds frequently invest in, to various degrees and combinations, illiquid exchange-traded assets or difficult-to-price over-the-counter securities (Asness, Krail, and Liew (2001)), a hedge fund manager can have great discretion in marking the portfolio's value at the end of each month to arrive at the fund's net asset value. Hedge funds' compensation scheme, especially the "high watermark" provision, does give managers an incentive to "smooth" their returns by making their portfolio to less than their actual value in months with large positive returns so as to create a "cushion" for those months with lower returns.

Managers have difficulty obtaining an accurate value of illiquid assets and most rely on observed transaction prices for similar assets. Similar to its effects in real estate property value, such partial adjustment or "smoothing" produces systematic valuation errors which tend not to be diversified away. It results in serial correlation in hedge fund returns and underestimation of their true standard deviations. Brooks and Kat (2001) adapt the approach proposed by Geltner (1993) to reconcile stale price problems in hedge fund returns. We follow their approach, outlined in Appendix A, to unsmooth hedge fund returns.

Our objective is to determine how much capital to allocate to each hedge fund strategy (rather than each specific hedge fund). Thus, we form seven hedge fund portfolio time series, each of which is equally weighted among individual hedge funds within that strategy. Tables 1 and 2 provide a statistical summary of the "smoothed" and "unsmoothed" returns.

4 Empirical results

4.1 Trade-off between multiple objectives

Table 3 clearly shows that different combinations of investors' preferences over expected return (α), skewness (β) and kurtosis (γ) lead to optimal hedge fund portfolios with substantially different moment characteristics. The more importance investors attach to a certain moment, i.e. the greater the preference parameter for this moment, the more favorable value of this moment statistic would be in the optimal portfolio. The tradeoff between the four moments implies that as one moment statistic improves, at least one of the other three moment statistics deteriorates. For example, the highest skewness is achieved in portfolio 3-A which has a higher preference parameter over skewness ($\beta = 3$), but also does not incorporate a preference for kurtosis ($\gamma = 0$). Consequently, the improvement of portfolio skewness is not restricted by the simultaneous requirement of improvement of portfolio kurtosis. The same pattern is observed for portfolio 3-B (with no preference over kurtosis) which has the highest expected return among all optimal portfolios in Table 3 and for portfolio 3-C (with no preference over skewness) which has lowest portfolio kurtosis among all optimal portfolios in Table 3.

We next investigate whether slightly changing just one of the preference parameters changes the optimal portfolio return characteristics in a predictable manner. For instance, we consider changing the preference parameters of portfolio 3-A from ($\alpha = 1, \beta = 3, \gamma = 0$) to ($\alpha = 2, \beta = 3, \gamma = 0$) while holding the values of standard deviation and kurtosis constant at their original optimal values. This results in the mean rising from 0.711 to 0.719 and the skewness falling from 0.883 to 0.879. Thus, as the investor preference for expected returns increases, he must settle for lower skewness, *ceteris paribus*. Similarly, we also consider changing the preference parameters of portfolio 3-C from ($\alpha = 1, \beta = 0, \gamma = 3$) to ($\alpha = 2, \beta = 0, \gamma = 3$) while holding the values of standard deviation and skewness constant. This results in the mean rising from 0.749 to 0.759 and the kurtosis rising from -0.252 to -0.249. Thus, as preference for expected returns increases, the investor must settle for higher kurtosis, holding skewness and standard deviation constant. Similar analysis confirms that when mean and standard deviation are held constant, higher preference for skewness leads to higher skewness, but also higher kurtosis. Therefore, expected return, skewness and kurtosis are conflicting objectives in portfolio diversification and risk adverse investors reward portfolios with high skewness and low kurtosis with a lower required rate of return.

In our approach, portfolio variance is scaled to one only on a relative basis and is thus essentially not bounded (unless specifically constrained). It also plays an important role in the tradeoff interaction. For example, compare portfolio 3-E ($\alpha = 2, \beta = 3, \gamma = 1$) with portfolio 3-G ($\alpha = 1, \beta = 2, \gamma = 3$). While the expected returns of the optimal portfolios are similar, the improvement of portfolio skewness in portfolio 3-E relative to portfolio 3-G comes at the sacrifice of portfolio standard deviation and kurtosis.

Table 4 investigates the interaction between expected return, skewness, and kurtosis, holding investor preferences constant, as portfolio standard deviation becomes increasingly constrained. For instance, in portfolio 4-A standard deviation is unconstrained, in portfolio 4-B standard deviation is constrained to be less than or equal to 90% of its unconstrained value, and in portfolio 4-C standard deviation is constrained to be less than or equal to 80% of its unconstrained value. We observe that the reduction in risk (from 4-A to 4-B and from 4-B to 4-C) comes from the deterioration in portfolio skewness and/or kurtosis, rather than in portfolio expected returns. Similarly, as portfolio risk is decreased from portfolio 4-D to 4-F or from portfolio 4-G to 4-I, portfolio skewness and/or kurtosis deteriorate, while portfolio expected returns increase. This is consistent with the possibility that correlation between the second, third, and fourth moments is higher than that between the first, third, and fourth moments.

4.2 Asset allocation for optimal hedge fund portfolios

Table 5 reports optimal allocation weights across different hedge fund strategies for different investor preferences over expected return, skewness, and kurtosis. Under all circumstances, equity market neutral funds and global macro funds are chosen. This is consistent with the finding of Davies, Kat and Lu (2003) that optimal multi-strategy hedge fund portfolios should include equity market neutral funds since they always produce low correlation, positive coskewness and low cokurtosis with funds from other strategies. Global macro funds also produce low cokurtosis with funds from other strategies. It is important to emphasize that global macro funds would *not* be included in the optimal fund of hedge funds portfolio if the analysis was limited to mean-variance space (because of their comparatively high embedded volatility). Thus, portfolios selected under the traditional mean-variance framework are not necessarily the same, and may be strictly inferior, to those selected under a mean-variance-skewness-kurtosis framework.

Despite the claim that long/short equity funds and distressed securities funds are return enhancers, our optimisation rarely allocates them capital. This is because in our four-moment framework, enhancing return is only one of three objectives. Long/short equity funds and distressed securities funds always produce negative coskewness and higher cokurtosis with funds from other strategies (see Davies, Kat and Lu (2003) for more details). A caveat for this is that in our data set, merger arbitrage funds have the highest mean return among all strategies, which may be due to their abnormally good performance in the sample period. Thus, when investors attach more importance on expected returns, such as in portfolios 5-C and 5-G, merger arbitrage funds are allocated more weight than otherwise.

We next consider whether our results are sensitive to constraints on standard deviation, skewness, and kurtosis. Part A of Table 6 illustrates optimal allocations for portfolios that have the same preferences over objectives as those in Table 5, but under the constraint that the portfolio standard deviation is less than 10% its unconstrained value. All portfolios in part A increase their capital loading on equity market neutral funds and decrease their capital loading on global macro funds, compared with their counterparts in Table 5. Thus, risk reduction is provided by equity market neutral funds, but not by global macro funds. Part B illustrates the case in which the value of each portfolio's skewness is constrained to be 10% higher than its counterpart in Table 5. The constraint on skewness causes capital loadings in global macro funds in all portfolios to rise, but capital loadings in equity market neutral funds to either remain the same or decrease, in comparison with their unconstrained counterparts. This suggests that skewness improvement can be attributed more to the portfolio weight in global macro funds than that in equity market neutral funds.

Part C of Table 6 illustrates how asset allocations change when constraints to portfolio kurtosis are imposed. Similar to constraints on standard deviations, the addition of constraints on kurtosis has increased capital loadings for equity market neutral funds. Note, however, that the changes in capital loadings are smaller than those reported in Parts A and B. Thus, constraints on portfolio kurtosis do not impact portfolio allocation as much as constraints on standard deviation and skewness. Capital loadings on equity market neutral funds either increase or remain the same while those on global macro funds either decrease or remain the same.

In summary, we have shown that equity market neutral funds and global macro funds have important roles in optimal hedge fund portfolios. Specifically, equity market neutral funds are risk and kurtosis reducers while global macro funds are skewness enhancers.

5 Conclusion

This paper has incorporated investor preferences into a PGP optimization function. This approach allows us to solve for multiple competing (and often conflicting) hedge fund allocation objectives within the mean-variance-skewness-kurtosis framework. Our empirical analysis shows that equity market neutral funds and global macro funds have predominant roles in optimal hedge fund portfolios. Specifically, equity market neutral funds are risk and kurtosis reducers while global macro funds are skewness enhancers.

Mixed-strategy funds of hedge funds are increasingly preferred over their single-strategy counterparts, because of the higher diversification benefit inherent in a more diverse mix. This trend suggests that allocation decisions across different fund strategy classes will become even more important in the future.

A Procedure to "unsmooth" data

The observed (or smoothed) value V_t^* of a hedge fund at time t can be expressed as a weighted average of the underlying (true) value at time t, V_t, and the smoothed value at time $t-1$, V_{t-1}^*:

$$V_t^* = \alpha V_t + (1-\alpha)V_{t-1}^*.$$

Let B be the backshift operator defined by $B^L x_t = x_{t-L}$. Define the following lag function, $L_t(\alpha)$, which is a polynomial B, with different coefficients for each of the 12 appraisal cohorts indexed by t

$$L_t(\alpha) = \frac{t}{12} + \sum_{L=1}^{\infty} \left[(1-\alpha)^{L-1} \left(\frac{12-t}{12} \right) + (1-\alpha)^L \left(\frac{t}{12} \right) \right] B^L.$$

Let r_t and r_t^* denote the true underlying (unobservable) return and the observed return at time t respectively. The monthly smoothed return is given by

$$r_t^* = \alpha L_m(\alpha) r_t.$$

It is possible to derive the following relationship from above:

$$\begin{aligned} r_t^* &= \alpha r_t + (1-\alpha) r_{t-1}^* \\ &= \alpha r_t + \alpha(1-\alpha) r_{t-1} + \alpha(1-\alpha)^2 r_{t-2} \cdots . \end{aligned}$$

Here we implicitly assume that hedge fund managers use a single exponential smoothing approach. This yields an unsmoothed series with zero first order autocorrelation:

$$r_t = \frac{r_t^* - (1-\alpha) r_{t-1}^*}{\alpha},$$

Since the stock market indices have around zero autocorrelation coefficients, it seems plausible in the context of the results above to set $1-\alpha$ equal to the first order autocorrelation coefficient. The newly constructed return series, r_t, has the same mean as r_t^*, and zero first order autocorrelation (aside from rounding errors), but with higher standard deviation.

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Table 1: **Statistical summary of individual hedge fund returns.** Reported values are calculated from monthly net-of-all-fee returns and averaged across funds. Kurtosis represents the excess kurtosis. First-order to fourth-order autocorrelation is given by AC(1)–AC(4).

	Mean	Std Dev	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)	AC(4)
Convertible arbitrage	0.96	3.01	-1.14	5.93	0.30	0.15	0.09	0.02
Distressed securities	0.89	2.37	-0.78	6.36	0.25	0.08	-0.04	0.02
Equity market neutral	0.54	2.70	-0.41	2.82	0.20	0.03	0.05	0.05
Global macro	0.77	5.23	1.06	7.63	0.11	0.01	-0.00	-0.03
Long/short equity	1.34	5.83	0.00	3.35	0.09	-0.00	0.01	-0.03
Merger arbitrage	1.17	1.75	-0.50	4.96	0.10	-0.00	0.00	-0.03
Emerging markets	0.22	7.85	-0.86	5.79	0.10	-0.01	-0.00	-0.02

Table 2: **Statistical summary of unsmoothed individual hedge fund returns.** Reported values are calculated from net-of-all-fee returns and averaged across funds. Kurtosis represents the excess kurtosis. First-order to fourth-order autocorrelation is given by AC(1)–AC(4).

	Mean	Std Dev	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)	AC(4)
Convertible arbitrage	0.96	3.99	-0.91	5.46	0.00	-0.03	-0.01	-0.02
Distressed securities	0.91	3.06	-0.67	6.60	0.01	-0.03	-0.01	-0.02
Equity market neutral	0.55	3.06	-0.39	2.94	0.01	-0.03	-0.01	-0.02
Global macro	0.76	5.35	1.03	7.16	0.01	-0.02	-0.01	-0.03
Long/short equity	1.37	6.35	0.01	3.19	0.00	-0.03	-0.00	-0.03
Merger arbitrage	1.17	2.06	-0.46	4.65	0.00	-0.03	-0.01	-0.02
Emerging markets	0.23	9.63	-0.91	5.91	0.01	-0.03	-0.01	-0.02

Table 3: **Moment statistics for optimal hedge fund portfolios.** Reported values are calculated from monthly net-of-all-fee returns. Hedge funds are selected from convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β) and excess kurtosis (γ).

Portfolio	3-A	3-B	3-C	3-D	3-E	3-F	3-G
α	1	3	1	3	2	1	1
β	3	1	0	2	3	3	2
γ	0	0	3	1	1	2	3
Mean	0.71	0.85	0.75	0.76	0.72	0.70	0.72
Std dev	2.06	1.02	1.12	1.23	1.52	1.52	1.43
Skew	0.88	-0.25	0.02	0.42	0.73	0.60	0.39
Kurtosis	0.97	0.73	-0.25	0.07	0.47	0.11	-0.14

Table 4: **Moment statistics for optimal fund of hedge fund portfolios with constrained standard deviation.** Reported values are calculated from monthly net-of-all-fee returns. Hedge funds are selected from convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β), and excess kurtosis (γ). In the second and third columns of each section, standard deviation is constrained to be less than or equal to 90% and 80%, respectively, of the unconstrained standard deviation ($\hat{\sigma}$) obtained in the first column.

Portfolio	4-A	4-B	4-C	4-D	4-E	4-F	4-G	4-H	4-I
α	3	3	3	2	2	2	1	1	1
β	2	2	2	3	3	3	3	3	3
γ	1	1	1	1	1	1	2	2	2
Constraint	none	$\sigma \leq 0.9\hat{\sigma}$	$\sigma \leq 0.8\hat{\sigma}$	none	$\sigma \leq 0.9\hat{\sigma}$	$\sigma \leq 0.8\hat{\sigma}$	none	$\sigma \leq 0.9\hat{\sigma}$	$\sigma \leq 0.8\hat{\sigma}$
Mean	0.76	0.77	0.79	0.72	0.73	0.74	0.71	0.71	0.73
Std dev	**1.23**	**1.11**	**0.98**	**1.52**	**1.37**	**1.22**	**1.52**	**1.37**	**1.22**
Skew	0.42	0.17	-0.27	0.73	0.61	0.41	0.60	0.57	0.44
Kurtosis	0.07	-0.06	0.10	0.47	0.26	0.03	0.11	0.10	0.00

Table 5: **Asset allocation for optimal fund of hedge fund portfolios.** Reported values are calculated from monthly net-of-all-fee returns. Hedge funds are selected from convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β), and excess kurtosis (γ). Mean-var corresponds to the portfolio which has the highest information ratio.

Portfolio	5-A	5-B	5-C	5-D	5-E
α	1	3	1	1	Mean-var
β	3	1	0	1	
γ	0	0	3	1	
Convertible arbitrage	0.00	0.00	0.09	0.07	0.00
Distressed securities	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.22	0.38	0.49	0.46	0.29
Global macro	0.78	0.21	0.24	0.32	0.00
Long/short equity	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	0.00	0.41	0.18	0.14	0.71
Emerging markets	0.00	0.00	0.00	0.00	0.00

	5-F	5-G	5-H	5-I	5-J
α	1	1	3	3	2
β	3	1	1	2	3
γ	1	3	1	1	1
Convertible arbitrage	0.02	0.09	0.04	0.01	0.00
Distressed securities	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.38	0.48	0.47	0.42	0.39
Global macro	0.59	0.27	0.22	0.35	0.51
Long/short equity	0.01	0.00	0.00	0.00	0.00
Merger arbitrage	0.00	0.16	0.27	0.22	0.10
Emerging markets	0.00	0.00	0.00	0.00	0.00

Table 6: **Asset allocation for optimal hedge fund portfolios with constrained portfolio standard deviation, skewness or kurtosis.** Reported values are calculated from monthly net-of-all-fee returns. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β), and excess kurtosis (γ), selected from convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Bold numbers indicate that the strategy capital loading has increased relative to the optimal unconstrained portfolio.

Portfolio	6-A	6-B	6-C	6-D	6-F	6-G	6-H	6-I	6-J
α	1	3	1	1	1	1	3	3	2
β	3	1	0	1	3	1	1	2	3
γ	0	0	3	1	1	3	1	1	1
A: Standard deviation constrained (10% improvement)									
Convertible arbitrage	0.00	0.00	0.04	0.04	0.00	0.05	0.00	0.00	0.00
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	**0.31**	**0.46**	**0.52**	**0.50**	**0.41**	**0.51**	**0.52**	**0.47**	**0.42**
Global macro	0.68	0.09	0.16	0.24	0.51	0.19	0.12	0.26	0.42
Long/short equity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	**0.02**	**0.45**	**0.27**	**0.22**	**0.07**	**0.24**	**0.35**	**0.26**	**0.15**
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B: Skewness constrained (10% improvement)									
Convertible arbitrage	0.00	0.00	0.09	0.07	0.00	0.09	0.04	0.01	0.00
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.18	0.38	0.49	0.46	0.28	0.48	0.47	0.42	0.35
Global macro	**0.82**	**0.21**	**0.24**	**0.33**	**0.72**	**0.27**	**0.22**	**0.36**	**0.58**
Long/short equity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	0.00	0.40	0.18	0.14	0.00	0.15	0.27	0.21	0.06
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C: Kurtosis constrained (10% improvement)									
Convertible arbitrage	0.00	0.00	**0.10**	**0.08**	**0.02**	0.09	**0.04**	**0.01**	**0.01**
Distressed securities	0.00	0.00	0.00	0.00	0.00	**0.11**	0.00	0.00	0.00
Equity market neutral	**0.29**	**0.39**	**0.49**	**0.47**	**0.39**	**0.54**	**0.47**	**0.43**	**0.39**
Global macro	0.71	**0.21**	**0.25**	0.31	0.57	0.25	**0.22**	0.35	0.49
Long/short equity	0.00	0.00	0.00	0.00	**0.02**	0.00	0.00	0.00	0.00
Merger arbitrage	0.00	0.40	0.16	**0.14**	0.00	0.00	0.27	0.22	0.10
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00